5 Using graphs

In this chapter you will learn how to:

- use the link between solving simultaneous equations and intersecting graphs
- determine a number of intersections between a line and a curve
- use transformations of graphs
- use graphs and applications of direct and inverse proportion
- illustrate two-variable inequalities on a graph.

Before you start…

Chapter 3	You should know how to solve quadratic equations.	1 Solve $x^2 + x - 1 = 0$.
Chapter 3	You should know how to use the quadratic discriminant to determine the number of real solutions of a quadratic equation.	2 How many real solutions are there to the equation $x^2 + 4x + 4 = 0$?
GCSE	You should know how to solve simple linear simultaneous equations by elimination.	3 Solve these simultaneous equations. $x + 2y = 5$ $3x + 4y = 11$
Chapter 2	You should know how to solve equations involving indices.	4 Solve $2^x = 8$.
GCSE	You should know how to solve linear inequalities.	5 Solve $3x + 1 > 13$.
GCSE	You should be able to find equations for direct and inverse proportion.	6 V is inversely proportional to the square of r. When $r = 2$, $V = 12$. Find an expression for V in terms of r.

Why use graphs?

Graphs are an alternative way of expressing a relationship between two variables. Understanding the connection between graphs and equations (or inequalities), and being able to switch between the two representations, gives you a much wider variety of tools to solve mathematical problems.

Section 1: Intersections of graphs

You already know how to solve linear simultaneous equations, and how to use simultaneus equations to find the point of intersection of two straight lines. You can apply similar ideas to find intersections between curves whose equations involve quadratic functions. Whenever you are finding an intersection between two graphs, you are solving simultaneous equations. This means that the values you find for x and y must satisfy both equations.

Gateway to A Level

For revision of linear simultaneous equations, see Gateway to A Level section J.

The intersection of two graphs can always be found using technology (for example, graphing software or a graphical calculator). However, this usually only gives approximate solutions. If you need exact solutions you have to use an algebraic method. In many cases the best method is **substitution**, where you replace every occurrence of one variable in one equation by its expression from the other equation.

WORKED EXAMPLE 5.1

Find the coordinates of the points of intersection of the line $y = 2x - 1$ and the parabola $y = x^2 - 3x + 5$.

$x^2 - 3x + 5 = 2x - 1$

At the intersection points, the y-coordinates for the two curves are equal, so you can replace y in the first equation with the expression for y from the second equation.

$x^2 - 5x + 6 = 0$

This is a quadratic equation. Try to factorise.

$(x - 2)(x - 3) = 0$

$x = 2$ or 3

$y = 2x - 1$

You also need to find the y-coordinates, by substituting back into one of the equations for y (both should give the same answer). Pick the first equation, as it is easier.

$x = 2: y = 2 \times 2 - 1 = 3$

$x = 3: y = 2 \times 3 - 1 = 5$

The coordinates are $(2, 3)$ and $(3, 5)$.

WORK IT OUT 5.1

Solve $x + y = 3$ and $y^2 + 2x^2 = 9$.

Which is the correct solution? Can you identify the errors made in the incorrect solutions?

Solution 1	Solution 2	Solution 3
Squaring $x + y = 3$ gives $x^2 + y^2 = 9$. Subtracting this from the second equation gives $x^2 = 0$ so $x = 0$. Substituting into the first equation, $y = 3$. Checking this in the second equation gives: $3^2 + 2 \times 0^2 = 9$	If $x + y = 3$ then $y = 3 - x$ Substituting into the second equation: $y^2 + 2x^2 = 9$ $(3 - x)^2 + 2x^2 = 9$ $9 - 6x + x^2 + 2x^2 = 9$ $3x^2 - 6x = 0$ Dividing by $3x$: $x - 2 = 0$ $x = 2$ Substituting into $y = 3 - x$: $x = 2, y = 1$	Rearranging the first equation gives $y = 3 - x$. Substituting into the second equation: $y^2 + 2x^2 = 9$ $(3 - x)^2 + 2x^2 = 9$ $9 - 6x + x^2 + 2x^2 = 9$ $3x^2 - 6x = 0$ $3x(x - 2) = 0$ $x = 0$ or 2 Substituting into the first equation: $x = 0, y = 3$ $x = 2, y = 1$

EXERCISE 5A

1 Find the coordinates of intersection of the given curve and the given straight line.

a **i** $y = x^2 + 2x - 3$ and $y = x - 1$ **ii** $y = x^2 - 4x + 3$ and $y = 2x - 6$

b **i** $y = -x^2 + 3x + 9$ and $2x - y = 3$ **ii** $y = x^2 - 2x + 8$ and $x - y = 6$

2 Solve the following simultaneous equations:

a **i** $x - 2y = 1,\ 3xy - y^2 = 8$ **ii** $x + 2y = 3,\ y^2 + 2xy + 9 = 0$

b **i** $xy = 3,\ x + y = 4$ **ii** $x + y + 8 = 0,\ xy = 15$

c **i** $x + y = 5,\ y = x^2 - 2x + 3$ **ii** $x - y = 4,\ y = x^2 + x - 5$

3 Find the coordinates of the points of intersection of $y = \frac{1}{x}$ and $y = 2x$.

4 Solve simultaneously:

$3^x + 2^x = 13 \qquad 3^x - 2^x = 5$

5 Solve simultaneously:

$y = 2^x \qquad 4^x + y = 72$

6 The sum of two numbers is 8 and their product is 9.75.

a Show that this information can be written as a quadratic equation.

b What are the two numbers?

7 Solve the equations $xy + x = 0,\ x^2 + y^2 = 4$.

8 The equations $y = (x - 2)(x - 3)^2$ and $y = k$ have one solution for all $k < m$. Find the largest value of m.

Section 2: The discriminant revisited

Sometimes you only want to know how many intersection points there are, rather than to find their actual coordinates. The discriminant can be used to determine the number of intersections.

WORKED EXAMPLE 5.2

Find the set of values of k for which the line with equation $x + y = k$ intersects the curve with equation $x^2 - 4x + y^2 + 6y = 12$ at two distinct points.

Line equation: $y = k - x$

Try finding the intersections in terms of k and see if that gives you any ideas.

Substitute into the equation:

$x^2 - 4x + (k - x)^2 + 6(k - x) = 12$

$\Rightarrow x^2 - 4x + k^2 - 2kx + x^2 + 6k - 6x = 12$

$\Rightarrow 2x^2 - (10 + 2k)x + k^2 + 6k = 12$

At the intersection points, the y-coordinates for the two curves are equal, so you can replace y in the second equation by the expression for y from the first equation.

$\Rightarrow 2x^2 - (10 + 2k)x + (k^2 + 6k - 12) = 0$

This is a quadratic equation, so write it with one side equal to zero.

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Two solutions $\therefore \Delta > 0$

You know that the discriminant tells you the number of solutions of a quadratic equation.

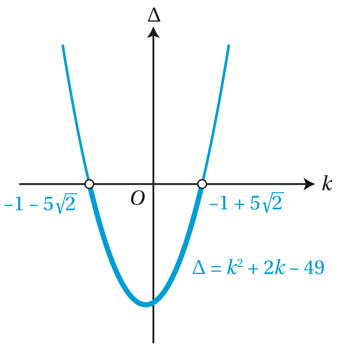
$\Delta = (10 + 2k)^2 - 8(k^2 + 6k - 12) > 0$

$\Rightarrow 100 + 40k + 4k^2 - 8k^2 - 48k + 96 > 0$

$\Rightarrow -4k^2 - 8k + 196 > 0$

$\Rightarrow k^2 + 2k - 49 < 0$

Divide both sides by −4. Remember that this reverses the inequality.



Roots: $k^2 + 2k - 49 = 0$

$k = \frac{-2 \pm \sqrt{4 + 4 \times 49}}{2}$

This is a quadratic inequality. To solve it, find where LHS = 0 and sketch the graph.

$= \frac{-2 \pm 2\sqrt{1 + 49}}{2}$

$= -1 \pm \sqrt{50} = -1 \pm 5\sqrt{2}$

The graph shows that the required interval is between the roots.

$\therefore -1 - 5\sqrt{2} < k < -1 + 5\sqrt{2}$

Tip

Questions which talk about the number of intersections are often solved using the discriminant.

Fast forward

The equation $x^2 - 4x + y^2 + 6y = 12$ in Worked example 5.2 actually represents a circle. You will study circles in Chapter 6.

EXERCISE 5B

1. Show that the line with equation $x - y = 6$ is a tangent to the curve with equation $x^2 - 6x + y^2 - 2y + 2 = 0$.
2. Find the exact values of m for which the line $y = mx + 3$ is a tangent to the curve with equation $y = 3x^2 - x + 5$.
3. Let C be the curve with equation $4x^2 + 9y^2 = 36$. Find the exact values of k for which the line $2x + 3y = k$ is a tangent to C.

Tip

A **tangent** touches the curve but does not cross at that point. With quadratic equations this means that there are repeated roots so the discriminant is zero. After studying Chapter 13 you will find another way of finding the tangent to a curve. However, this type of question is still best done using the discriminant.

4 Find the values of a for which the curve $y = x^2$ never touches the curve $y = a - (x - a)^2$.

5 Show algebraically that the line $y = kx + 5$ intersects the parabola $y = x^2 + 2$ twice for all values of k.

Section 3: Transforming graphs

From previous study you may know how changing the function changes the graph as summarised in Key point 5.1.

Key point 5.1

Transformation of $y = f(x)$	Transformation of graph
$y = f(x) + c$	Translation c units up.
$y = f(x + d)$	Translation d units to the *left*.
$y = pf(x)$	Vertical stretch, scale factor p
$y = f(qx)$	Horizontal stretch factor $\frac{1}{q}$
$y = -f(x)$	Reflection in the x-axis
$y = f(-x)$	Reflection in the y-axis

Tip

When c is negative the translation is down, and when d is negative it is to the right. When p or q are negative, the stretch is combined with a reflection.

Vertical transformations behave as expected, but the horizontal ones can be counter-intuitive; for example, $f(x + 3)$ translates the graph to the left.

The following proof shows why that is the case. However, you can use all the results from Key Point 5.1 without proof.

PROOF 2

Prove that the graph of $f(x + d)$ is a translation of the graph of $y = f(x)$, by d units to the left.

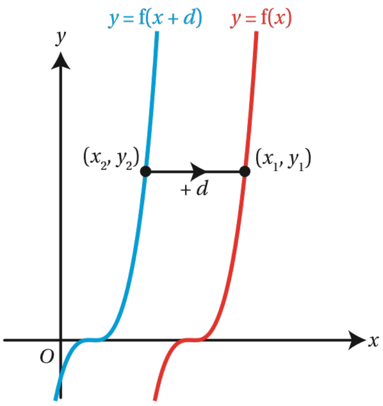
Let (x_1, y_1) be a point on the graph of $y = f(x)$ and (x_2, y_2) a point on the graph of $y = f(x + d)$.

Define variables. You have to be very careful and **not** assume that x's are all the same or y's are all the same.

When $x_1 = x_2 + d$ then:

$y_1 = f(x_1) = f(x_2 + d) = y_2$

This says that if two points are d units apart horizontally, then they are at the same height.



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Hence (x_2, y_2) is d units to the left of (x_1, y_1), and so the graph of $y = \mathrm{f}(x)$ is translated d units to the left to get the graph of $y = \mathrm{f}(x + d)$.

Interpret your calculation geometrically and write a conclusion.

WORKED EXAMPLE 5.3

The graph of $y = x^2 + 2x$ is translated 5 units to the left. Find the equation of the resulting graph in the form $y = ax^2 + bx + c$.

If $\mathrm{f}(x) = x^2 + 2x$, then the new graph is $y = \mathrm{f}(x + 5)$,

Relate the transformation to function notation.

$y = (x + 5)^2 + 2(x + 5)$
$= x^2 + 12x + 35$

Replace all x by $(x + 5)$ in the equation for the function.

Tip

A translation 5 units to the left could also be described in vector notation as $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$.

WORKED EXAMPLE 5.4

Describe a transformation which transforms the graph of $y = x^2 + 3x$ to the graph of $y = 4x^2 + 6x$.

Let $\mathrm{f}(x) = x^2 + 3x$

Then $4x^2 + 6x = (2x)^2 + 3(2x) = \mathrm{f}(2x)$

Try to relate the two equations by writing the second function in a similar way to the first.

It is a horizontal stretch with scale factor $\frac{1}{2}$.

Relate the function notation to the transformation.

WORKED EXAMPLE 5.5

The graph of $y = \mathrm{f}(x)$ has a single maximum point with coordinates (4, –3). Find the coordinates of the maximum point on the graph of $y = \mathrm{f}(-x)$.

The transformation taking $y = \mathrm{f}(x)$ to $y = \mathrm{f}(-x)$ is a reflection in the y-axis.

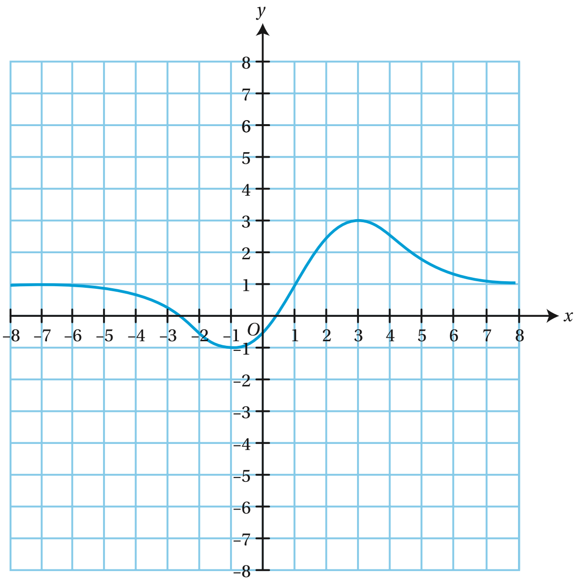
Relate the function notation to the transformation.

The maximum point is (–4, –3).

Reflection in the y-axis leaves y-coordinates unchanged and changes x to $-x$.

EXERCISE 5C

1 The graph of $y = \mathrm{f}(x)$ is shown. Sketch the graph of the following functions, including the position of the minimum and maximum points.



a **i** $y = \mathrm{f}(x) + 3$ **ii** $y = \mathrm{f}(x) + 5$

b **i** $y = \mathrm{f}(x) - 7$ **ii** $y = \mathrm{f}(x) - 0.5$

c **i** $y = \mathrm{f}(x + 2)$ **ii** $y = \mathrm{f}(x + 4)$

d **i** $y = \mathrm{f}(x - 1.5)$ **ii** $y = \mathrm{f}(x - 2)$

e **i** $y = 3\mathrm{f}(x)$ **ii** $y = 5\mathrm{f}(x)$

f **i** $y = \frac{\mathrm{f}(x)}{4}$ **ii** $y = \frac{\mathrm{f}(x)}{2}$

g **i** $y = \mathrm{f}(2x)$ **ii** $y = \mathrm{f}(6x)$

h **i** $y = \mathrm{f}\left(\frac{2x}{3}\right)$ **ii** $y = \mathrm{f}\left(\frac{5x}{6}\right)$

i **i** $y = -\mathrm{f}(x)$ **ii** $y = \mathrm{f}(-x)$

2 Find the equation of each of these graphs after the given transformation is applied:

a **i** $y = 3x^2$ after a translation of 3 units vertically up

ii $y = 9x^3$ after a translation of 7 units vertically down

b **i** $y = 7x^3 - 3x + 6$ after a translation of 2 units down

ii $y = 8x^2 - 7x + 1$ after a translation of 5 units up

c **i** $y = 4x^2$ after a translation of 5 units to the right

ii $y = 7x^2$ after a translation of 3 units to the left

d **i** $y = 3x^3 - 5x^2 + 4$ after a translation of 4 units to the left

ii $y = x^3 + 6x + 2$ after a translation of 3 units to the right

Elevate

To explore how transformations are related to the symmetries of a graph, see Extension sheet 5.

3 Find the required translations:

a **i** transforming the graph $y = x^2 + 3x + 7$ to the graph $y = x^2 + 3x + 2$

ii transforming the graph $y = x^3 - 5x$ to the graph $y = x^3 - 5x - 4$

b **i** transforming the graph $y = x^2 + 2x + 7$ to the graph $y = (x + 1)^2 + 2(x + 1) + 7$

ii transforming the graph $y = x^2 + 5x - 2$ to the graph $y = (x + 5)^2 + 5(x + 5) - 2$

c **i** transforming the graph $y = \sqrt{2x}$ to the graph $y = \sqrt{2x + 6}$

ii transforming the graph $y = \sqrt{2x + 1}$ to the graph $y = \sqrt{2x - 3}$

Elevate

For more examples like this, see Support sheet 5.

4 Find the equation of the graph after the given transformation is applied.

a **i** $y = 3x^2$ after a vertical stretch factor 7 relative to the x-axis.

ii $y = 9x^3$ after a vertical stretch factor 2 relative to the x-axis.

b **i** $y = 7x^3 - 3x + 6$ after a vertical stretch factor $\frac{1}{3}$ relative to the x-axis.

ii $y = 8x^2 - 7x + 1$ after a vertical stretch factor $\frac{4}{5}$ relative to the x-axis.

c **i** $y = 4x^2$ after a horizontal stretch factor 2 relative to the y-axis.

ii $y = 7x^2$ after a horizontal stretch factor 5 relative to the y-axis.

d **i** $y = 3x^3 - 5x^2 + 4$ after a horizontal stretch factor $\frac{1}{2}$ relative to the y-axis.

ii $y = x^3 + 6x + 2$ after a horizontal stretch factor $\frac{2}{3}$ relative to the y-axis.

5 Describe the following stretches:

a **i** transforming the graph $y = x^2 + 3x + 7$ to the graph $y = 4x^2 + 12x + 28$

ii transforming the graph $y = x^3 - 5x$ to the graph $y = 6x^3 - 30x$

b **i** transforming the graph $y = x^2 + 2x + 7$ to the graph $y = (3x)^2 + 2(3x) + 7$

ii transforming the graph $y = x^2 + 5x - 2$ to the graph $y = (4x)^2 + 5(4x) - 2$

c **i** transforming the graph $y = \sqrt{4x}$ to the graph $y = \sqrt{12x}$

ii transforming the graph $y = \sqrt{2x+1}$ to the graph $y = \sqrt{x+1}$

6 Find the equation of the graph after the given transformation is applied.

a **i** $y = 3x^2$ after reflection in the x-axis.

ii $y = 9x^3$ after reflection in the x-axis.

b **i** $y = 7x^3 - 3x + 6$ after reflection in the x-axis.

ii $y = 8x^2 - 7x + 1$ after reflection in the x-axis.

c **i** $y = 4x^2$ after reflection in the y-axis.

ii $y = 7x^3$ after reflection in the y-axis.

d **i** $y = 3x^3 - 5x^2 + 4$ after reflection in the y-axis.

ii $y = x^3 + 6x + 2$ after reflection in the y-axis.

7 Describe the following transformations:

a **i** transforming the graph $y = x^2 + 3x + 7$ to the graph $y = -x^2 - 3x - 7$

ii transforming the graph $y = x^3 - 5x$ to the graph $y = 5x - x^3$

b **i** transforming the graph $y=x^2+2x+7$ to the graph $y=x^2-2x+7$

ii transforming the graph $y=x^2-5x-2$ to the graph $y=x^2+5x-2$

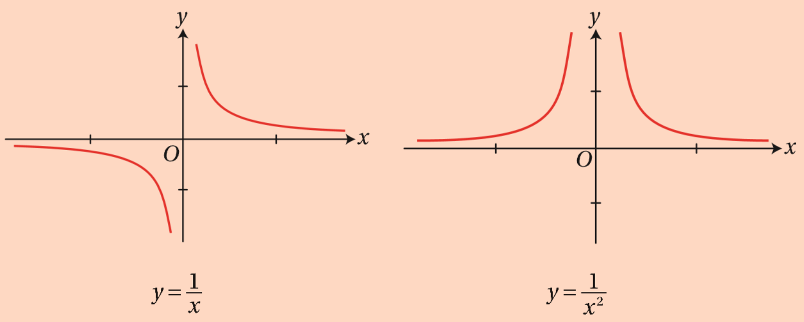
c **i** transforming the graph $y=\sqrt{4x}$ to the graph $y=\sqrt{-4x}$

ii transforming the graph $y=\sqrt{2x-1}$ to the graph $y=\sqrt{-1-2x}$

Section 4: Graphs of $\frac{a}{x}$ and $\frac{a}{x^2}$

You need to be able to sketch the graphs of $y=\frac{1}{x}$ and $y=\frac{1}{x^2}$.

Key point 5.2



The graphs of $\frac{a}{x}$ and $\frac{a}{x^2}$ are very similar to the graphs in Key point 5.2. They are vertically stretched by a factor of a.

Both graphs have two **asymptotes**. An asymptote is a line to which the curve gets closer and closer but never meets. These curves have asymptotes at $x=0$ and $y=0$.

EXERCISE 5D

1 **a** Write down the transformation that changes the graph of $y=\frac{a}{x}$ into the graph of $y=\frac{a}{x-d}$.

b Hence write down the equations of the asymptotes of the graph $y=\frac{a}{x-d}$.

2 Show that the curve $y=\frac{a}{x^2}$ is a horizontal stretch of the curve $y=\frac{1}{x^2}$ and find the stretch factor.

3 **a** Show that the curves $y=\frac{a}{x}$ and $y=\frac{b}{x^2}$ always intersect at exactly one point, P, and find the coordinates of that point.

b The origin and P are opposite vertices of a rectangle with sides parallel to the coordinate axes. Show that the area of this rectangle is independent of b.

4 Find a condition on m in terms of a and c so that the line $y = mx + c$ is a tangent to the curve $y = \frac{a}{x}$.

5 The function f(x) is a cubic polynomial. Show graphically that the curve $y = \frac{1}{x}$ can intersect this curve in 0, 1, 2, 3 or 4 places.

Section 5: Direct and inverse proportion

Direct proportion means that the ratio of two quantities is constant.

For example, if y is proportional to x^2 you can write $\frac{y}{x^2} = k$ or $y = kx^2$.

Inverse proportion means that the product of two quantities is constant.

For example, if y is inversely proportional to x^2 you write $yx^2 = k$ or $y = \frac{k}{x^2}$.

You can use your knowledge of graphs to sketch the graphs of two quantities if you are given information about their proportionality.

Linear functions are closely related to direct proportion: if $y = mx + c$ then $(y - c)$ is directly proportional to x.

Straight-line graphs can be used to represent or model a variety of real-life situations. In some situations, the linear model is only approximate. When making predictions, you should consider its accuracy and limitations.

Gateway to A Level

For a reminder of calculations involving direct and inverse proportion, see Gateway to A Level section K.

Fast forward

A common example where a straight line is used to make predictions is the line of best fit used in statistics. You will learn more about lines of best fit in Chapter 16, Section 4.

WORKED EXAMPLE 5.6

It takes me 12 minutes to drive from my house to the motorway. On the motorway, I drive at an average speed of 65 miles per hour.

a Approximately how long does it take me to drive to Leeds, which is 154 miles away?
b Write down an equation modelling the time, t hours, it takes me to drive to a city d miles away.
c Explain why this model only gives approximate times.

a Time in hours:

$0.2 + \frac{154}{65} = 2.57$ hours

(about 2 h 34 minutes)

Use time on motorway $= \frac{\text{distance}}{\text{speed}}$.

12 minutes $= 0.2$ hours.

b $t = 0.2 + \frac{d}{65}$

c The speed on the motorway is not constant.

You are modelling the speed as constant, although in reality this is not the case.

It doesn't take into account the time from getting off the motorway in Leeds.

The 154 miles distance is probably not exact; it doesn't specify where in Leeds you are going or exactly where it is measured from.

The speeds and distances quoted are probably only correct to the nearest integer.

All these considerations mean that the model does not give an exact answer, but it is probably good enough to be practical.

EXERCISE 5E

1 **a** If y is proportional to x^2 and $y = 12$ when $x = 2$, find the value of y when $x = 4$.

b Sketch the graph of y against x.

2 **a** If y is proportional to $x - 4$ and $y = 1$ when $x = 6$, find y when $x = 8$.

b Sketch the graph of y against x.

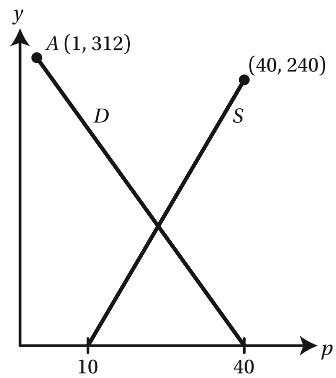
3 **a** If y is inversely proportional to x^2 and $y = 20$ when $x = 1$, find y when $x = 4$.

b Sketch the graph of y against x.

4 **a** If x is inversely proportional to $x + 1$ and $y = 9$ when $x = 3$, find y when $x = 5$.

b Sketch the graph of y against x.

5 Economists use supply and demand curves to model the number of items produced and sold at a particular price. Let £ p be the price of one item. Demand (D) is the number of items that can be sold at this price. Supply (S) is the number of items that the producer will make. The graph shows supply and demand in the simplest model, where both vary linearly with price.



a Show that the equation of line D is $y_D = 320 - 8p$ and find the equation of line S.

b What does the value 320 in the equation of D represent? Suggest why it may not be reasonable to extend the straight line for D beyond point A.

c What is the maximum price that can be charged before there is no more demand?

The market is said to be in equilibrium when supply equals demand.

d Find the equilibrium price of one item.

6 A provider offers two different mobile phone contracts:

A The set-up cost of £65, plus calls at 3 p per minute.

B No set-up costs, calls cost 5 p per minute.

a Write down an equation for the total cost, £C, of making m minutes of calls for each contract.

b Hence find after how many minutes of calls contract A becomes better value.

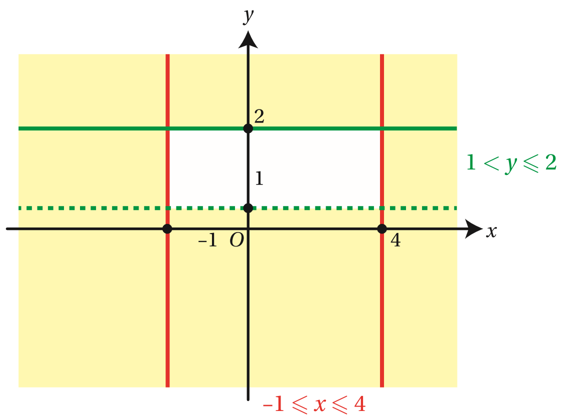
7 y is inversely proportional to x^2 and z is inversely proportional to y. Sketch a graph of z against x.

8 The strength of the Earth's gravitational field is inversely proportional to the square of the distance from the centre of the Earth. If a satellite is put into orbit, the distance to the centre of the Earth is increased by 10%. Find the percentage decrease in the gravitational field strength.

Section 6: Sketching inequalities in two variables

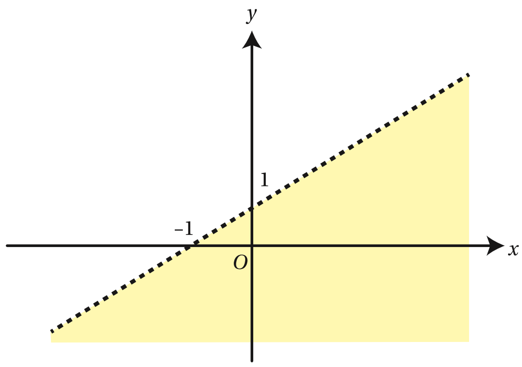
You can represent inequalities in one variable on a number line.
For example, the inequality $1 < x < 4$ can be represented by:

If there is also another variable, you can represent the inequality on a graph:



In this graph you can see the convention that the part that satisfies the inequality is left unshaded. This is so that when you have several inequalities on one graph, the region which satisfies all the inequalities is clear.

If the inequality involves both variables you can still represent the solution by shading. For example, $y > x + 1$ is shown on the following graph. The required region has been left unshaded.



Notice that since the line $y = x + 1$ is not included it is drawn as a dashed line.

Key point 5.3

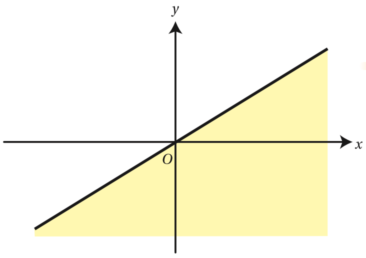
The general process for illustrating inequalities is:

- draw the associated equation on the graph, using a dashed line if the curve is not included
- test a convenient point on one side of the curve
- shade the side that does not satisfy the inequality.

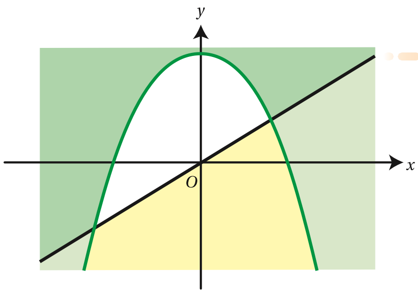
WORKED EXAMPLE 5.7

a Represent the inequalities $y \geqslant x$ and $y \leqslant 1 - x^2$ on a graph.
b Find the largest value of x that satisfies these inequalities.

a



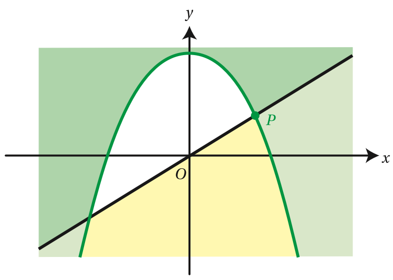
First sketch $y = x$. It is a solid line since the line is included in the inequality. You can try the point (1, 0) and it does not satisfy the inequality, so you shade that side of the line.



Then sketch $y = 1 - x^2$. This is a solid line since the line is included in the inequality. You can try the point (0, 0) and it does satisfy the inequality, so you shade the other side of the curve.

b The largest x value corresponds to the point labelled P.

You need to find the point in the unshaded region with the largest x value.



P occurs where:

To find this point you use simultaneous equations.

$x = 1 - x^2$

So $x = \dfrac{-1 \pm \sqrt{5}}{2}$

You can solve this using the quadratic formula.

So the largest value of x is $\dfrac{-1+\sqrt{5}}{2}$

The larger solution is the one with a plus.

EXERCISE 5F

1. Illustrate the following inequalities on a graph.

   a **i** $y > 1 + 2x$ **ii** $y < 2 + x$

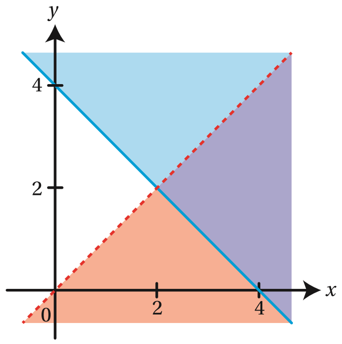
   b **i** $y + x \geqslant 1$ **ii** $y + 2x \geqslant 4$

   c **i** $y > x^2$ **ii** $y > -x^2$

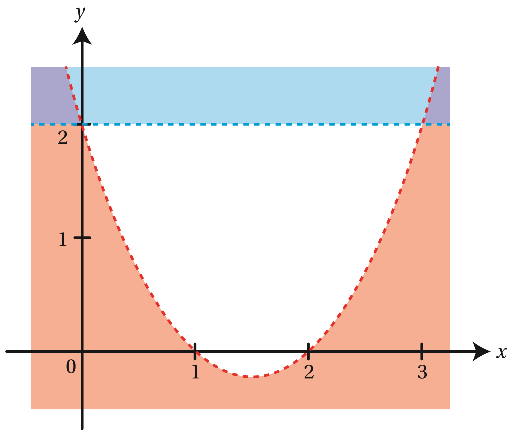
   d **i** $y > x^2 + 3x + 2$ **ii** $y > x^2 - 7x + 6$

   e **i** $y \leqslant x^2 + 2x + 1$ **ii** $y \leqslant x^2 - 7x + 10$

2. Illustrate the region $x > 0$, $y > 0$, $x + y < 4$ on a graph.
3. Illustrate the region $y \geqslant x^2$, $y < 4$ on a graph.
4. Illustrate the region $y > x^2 - 4x$ and $y < 2x - x^2$ on a graph.
5. Describe using inequalities the unshaded region in this graph.



6. This region is bounded by a parabola and a straight line. Describe using inequalities the unshaded region in this graph.



7. Find the largest integer value of x which satisfies $y < 120x - 2x^2$ and $y > 11x$.
8. Sketch $y > xy$.

Explore

Graphs of inequalities are needed to solve problems about maximising profits or minimising production time. Find out about a technique called **linear programming**.

Checklist of learning and understanding

- You can use substitution to solve simultaneous equations, which allows you to find the intersection point of two curves.
- The number of intersections of a quadratic curve and a straight line can be determined using the quadratic discriminant.
- Transforming a function results in a transformation of the graph of the function.

Transformation of $y = f(x)$	Transformation of graph
$y = f(x) + c$	Translation c up.
$y = f(x + d)$	Translation d to the **left**.
$y = pf(x)$	Vertical stretch, scale factor p relative to the x-axis
$y = f(qx)$	Horizontal stretch factor $\frac{1}{q}$ relative to the y-axis
$y = -f(x)$	Reflection in the x-axis
$y = f(-x)$	Reflection in the y-axis

- You should be able to sketch the graphs of $y = \frac{a}{x}$ and $y = \frac{a}{x^2}$.
- You should be able to interpret descriptions of the proportionality of two variables and to sketch the associated graph. You should also be able to use a linear model in a variety of contexts and understand that models sometimes only give approximate predictions.
- You can represent inequalities in two variables graphically by shading.

Mixed practice 5

1. Find the intersection of the graphs $x^2 + y^2 = 25$ and $x + y = 7$.

2. **a** Illustrate the region represented by the inequalities $x + y < 3$, $y \geqslant 0$, $y < 2x$.

   b Find the upper bound for the values of y that satisfy these inequalities.

3. Find the transformation that transforms the graph of $y = (x - 1)^2$ to the graph of $y = (x + 2)^2$.

4. If z is proportional to x^2 sketch the graph of z against x.

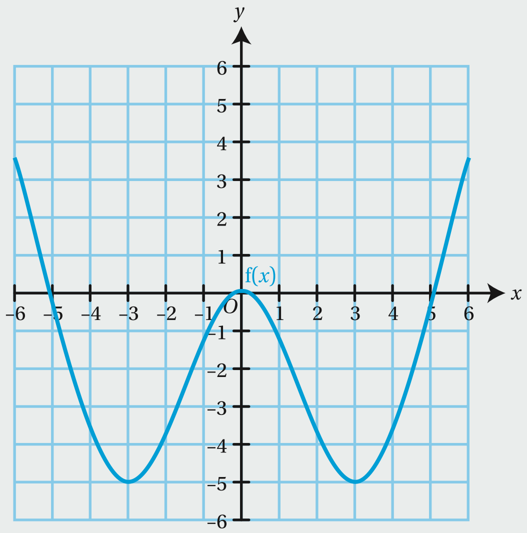
5. Two taxi companies have the following pricing structures:

   Company A charges £1.60 per kilometre.

   Company B charges £1.20 per kilometre plus £1.50 call-out charge.

   Find the length of the journey for which the two companies charge the same amount.

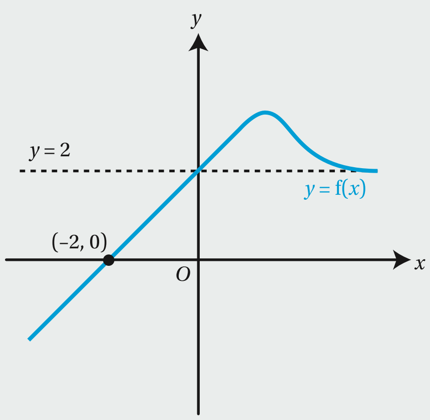
6. The graph of $y = f(x)$ is shown.



   a Sketch the graph of $y = f(x - 1) + 2$.

   b State the coordinates of the maximum point of the new graph.

7. The diagram shows a part of the graph of $y = f(x)$.



   Sketch the graph of $y = f(3x)$.

8 **a** The curve $y = x^2$ is translated 2 units in the positive x direction. Find the equation of the curve after it has been translated.

b The curve $y = x^3 - 4$ is reflected in the x-axis. Find the equation of the curve after it has been reflected.

9 A doctor thinks that mass of a baby can be modelled as a linear function of age. A particular baby had a mass of 4.1 kg aged 2 weeks, and 4.8 kg aged 5 weeks.

a If M is the mass of the baby aged n weeks, show that the straight line model results in the equation $M = 0.233n + 3.63$, where the coefficients have been rounded to three significant figures.

b Give an interpretation of the values 0.233 and 3.63 in the equation in part **a**.

c The normal mass of a healthy one-year-old baby is approximately between 10 and 12 kg. Is the linear model appropriate for babies as old as one year?

10 **a** Solve the simultaneous equations
$y = 2x^2 - 3x - 5 \qquad 10x + 2y + 11 = 0$

b What can you deduce from the answer to part **a** about the curve $y = 2x^2 - 3x - 5$ and the line $10x + 2y + 11 = 0$?

11 Given that x is inversely proportional to y and z is proportional to x^2 sketch the graph of z against y.

12 **a** By using an appropriate substitution find the exact solutions to the equation
$x^4 + 36 = 13x^2$

b Hence solve the inequality $x^4 + 36 \leqslant 13x^2$.