# Compound Angle Formulae Exam Questions (From OCR 4723)

**Q1, (Jan 2009, Q9)**

**(i)** By first expanding $\cos(2\theta + \theta)$, prove that

$$\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta. \qquad [4]$$

**(ii)** Hence prove that

$$\cos 6\theta \equiv 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1. \qquad [3]$$

**(iii)** Show that the only solutions of the equation

$$1 + \cos 6\theta = 18\cos^2\theta$$

are odd multiples of $90^\circ$. **[5]**

---

**Q2, (Jan 2010, Q9)**

The value of $\tan 10^\circ$ is denoted by $p$. Find, in terms of $p$, the value of

**(i)** $\tan 55^\circ$, **[3]**

**(ii)** $\tan 5^\circ$, **[4]**

**(iii)** $\tan\theta$, where $\theta$ satisfies the equation $3\sin(\theta + 10^\circ) = 7\cos(\theta - 10^\circ)$. **[5]**

---

**Q3, (Jun 2011, Q9)**

**(i)** Prove that $\dfrac{\sin(\theta - \alpha) + 3\sin\theta + \sin(\theta + \alpha)}{\cos(\theta - \alpha) + 3\cos\theta + \cos(\theta + \alpha)} \equiv \tan\theta$ for all values of $\alpha$. **[5]**

**(ii)** Find the exact value of $\dfrac{4\sin 149^\circ + 12\sin 150^\circ + 4\sin 151^\circ}{3\cos 149^\circ + 9\cos 150^\circ + 3\cos 151^\circ}$. **[3]**

**(iii)** It is given that $k$ is a positive constant. Solve, for $0^\circ < \theta < 60^\circ$ and in terms of $k$, the equation

$$\frac{\sin(6\theta - 15^\circ) + 3\sin 6\theta + \sin(6\theta + 15^\circ)}{\cos(6\theta - 15^\circ) + 3\cos 6\theta + \cos(6\theta + 15^\circ)} = k. \qquad [4]$$

---

**Q4, (Jan 2013, Q9)**

**(i)** Prove that

$$\cos^2(\theta + 45^\circ) - \tfrac{1}{2}(\cos 2\theta - \sin 2\theta) \equiv \sin^2\theta. \qquad [4]$$

**(ii)** Hence solve the equation

$$6\cos^2(\tfrac{1}{2}\theta + 45^\circ) - 3(\cos\theta - \sin\theta) = 2$$

for $-90^\circ < \theta < 90^\circ$. **[3]**

**(iii)** It is given that there are two values of $\theta$, where $-90^\circ < \theta < 90^\circ$, satisfying the equation

$$6\cos^2(\tfrac{1}{3}\theta + 45^\circ) - 3(\cos\tfrac{2}{3}\theta - \sin\tfrac{2}{3}\theta) = k,$$

where $k$ is a constant. Find the set of possible values of $k$. **[3]**

---

**Q5, (Jun 2015, Q9)**

It is given that $f(\theta) = \sin(\theta + 30°) + \cos(\theta + 60°)$.

**(i)** Show that $f(\theta) = \cos\theta$. Hence show that

$$f(4\theta) + 4f(2\theta) \equiv 8\cos^4\theta - 3.$$ **[6]**

**(ii)** Hence

**(a)** determine the greatest and least values of $\dfrac{1}{f(4\theta) + 4f(2\theta) + 7}$ as $\theta$ varies, **[3]**

**(b)** solve the equation

$$\sin(12\alpha + 30°) + \cos(12\alpha + 60°) + 4\sin(6\alpha + 30°) + 4\cos(6\alpha + 60°) = 1$$

for $0° < \alpha < 60°$. **[4]**

---

**Q6, (Jan 2006, Q9)**

**(i)** By first writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$ [4]

**(ii)** Determine the greatest possible value of

$$9\sin\left(\tfrac{10}{3}\alpha\right) - 12\sin^3\left(\tfrac{10}{3}\alpha\right),$$

and find the smallest positive value of $\alpha$ (in degrees) for which that greatest value occurs. [3]

---

## Solving Equations Using Compound Angle Formulae Exam Questions (From OCR 4754A)

**Q1, (Jun 2005, Q5)**

Solve the equation $2\cos 2x = 1 + \cos x$, for $0° \leqslant x < 360°$. [7]

---

**Q2, (Jan 2006, Q4)**

Solve the equation $2\sin 2\theta + \cos 2\theta = 1$, for $0° \leqslant \theta < 360°$. [6]

---

**Q3, (Jun 2006, Q3)**

Given that $\sin(\theta + \alpha) = 2\sin\theta$, show that $\tan\theta = \dfrac{\sin\alpha}{2 - \cos\alpha}$.

Hence solve the equation $\sin(\theta + 40°) = 2\sin\theta$, for $0° \leqslant \theta \leqslant 360°$. [7]

---

**Q4, (Jan 2007, Q3)**

**(i)** Use the formula for $\sin(\theta + \phi)$, with $\theta = 45°$ and $\phi = 60°$, to show that $\sin 105° = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$. [4]

**(ii)** In triangle ABC, angle BAC = 45°, angle ACB = 30° and AB = 1 unit (see Fig. 3).

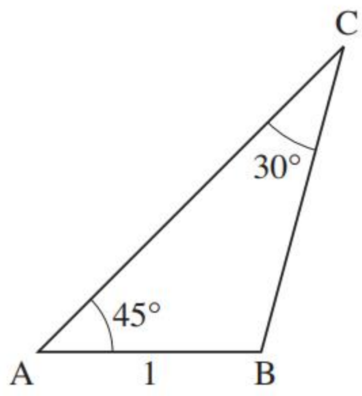



**Fig. 3**

Using the sine rule, together with the result in part **(i)**, show that $AC = \dfrac{\sqrt{3}+1}{\sqrt{2}}$. [3]

---

**Q5, (Jan 2008, Q4)**

The angle $\theta$ satisfies the equation $\sin(\theta + 45°) = \cos\theta$.

**(i)** Using the exact values of $\sin 45°$ and $\cos 45°$, show that $\tan\theta = \sqrt{2} - 1$. [5]

**(ii)** Find the values of $\theta$ for $0° < \theta < 360°$. [2]

---

**Q6, (Jun 2012, Q5)**

Given the equation $\sin(x + 45°) = 2\cos x$, show that $\sin x + \cos x = 2\sqrt{2}\cos x$.

Hence solve, correct to 2 decimal places, the equation for $0° \leqslant x \leqslant 360°$. **[6]**

---

***Q7, (Jun 2013, Q3)***

Using appropriate right-angled triangles, show that $\tan 45° = 1$ and $\tan 30° = \frac{1}{\sqrt{3}}$.

Hence show that $\tan 75° = 2 + \sqrt{3}$. **[7]**

---

***Q8, (Jun 2015, Q2)***

Express $6\cos 2\theta + \sin\theta$ in terms of $\sin\theta$.

Hence solve the equation $6\cos 2\theta + \sin\theta = 0$, for $0° \leqslant \theta \leqslant 360°$. **[7]**

---

## Small Angle Approximations Exam Questions

**Q1, (OCR H240/03, Sample Question Paper, Q4)**

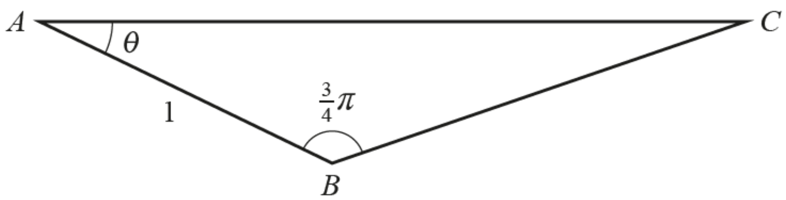
Show that, for a small angle θ , where θ is in radians,

$$1 + \cos\theta - 3\cos^2\theta \approx -1 + \frac{5}{2}\theta^2$$

**[4]**

---

**Q2, (OCR H240/03, Practice Paper Set 1, Q3)**




The diagram shows triangle $ABC$, in which angle $A = \theta$ radians, angle $B = \frac{3}{4}\pi$ radians and $AB = 1$ unit.

**(i)** Use the sine rule to show that $AC = \frac{1}{\cos\theta - \sin\theta}$. **[3]**

**(ii)** Given that $\theta$ is a small angle, use the result in part **(i)** to show that

$$AC \approx 1 + p\theta + q\theta^2,$$

where $p$ and $q$ are constants to be determined. **[4]**

---

**Q3, (OCR H240/02, Practice Paper Set 3, Q3)**

Use small angle approximations to estimate the solution of the equation $\frac{\cos\frac{1}{2}\theta}{1+\sin\theta} = 0.825$, if $\theta$ is small enough to neglect terms in $\theta^3$ or above. **[4]**