**1** Differentiate the following:

a  i  $y = x^4$        ii  $y = x^6$

b  i  $y = 3x^7$        ii  $y = -4x^5$

c  i  $y = -x$        ii  $y = 3x$

d  i  $y = 10$        ii  $y = -3$

e  i  $y = \dfrac{1}{3}x^6$        ii  $y = -\dfrac{3}{4}x^2$

f  i  $y = 4x^3 - 5x^2 + 2x - 8$        ii  $y = 2x^4 + 3x^3 - x$

g  i  $y = 7x - \dfrac{x^3}{2}$        ii  $y = 2 - 5x^4 + \dfrac{x^5}{5}$

**2** Find $f'(x)$ for the following functions:

a  i  $f(x) = x^{\frac{3}{2}}$        ii  $f(x) = x^{\frac{2}{3}}$

b  i  $f(x) = 8x^{\frac{1}{2}}$        ii  $f(x) = 6x^{\frac{4}{3}}$

c  i  $f(x) = \dfrac{4}{9}x^{\frac{3}{4}}$        ii  $f(x) = \dfrac{3}{5}x^{\frac{5}{6}}$

d  i  $f(x) = 3x^4 - 15x^{\frac{2}{5}} - 2$        ii  $f(x) = x^3 - \dfrac{3}{5}x^{\frac{5}{3}} + \dfrac{4}{3}x^{\frac{1}{2}}$

e  i  $f(x) = x^{-1}$        ii  $f(x) = x^{-3}$

f  i  $f(x) = x^{-\frac{1}{2}}$        ii  $f(x) = x^{-\frac{2}{3}}$

g  i  $f(x) = -6x^{-\frac{4}{3}}$        ii  $f(x) = -8x^{-\frac{3}{4}}$

h  i  $f(x) = 5x - \dfrac{8x^{-\frac{5}{2}}}{15}$        ii  $f(x) = -\dfrac{7x^{-\frac{3}{7}}}{3} + \dfrac{4x^{-6}}{3}$

**3** Find $\dfrac{dy}{dx}$ for the following:

a  i  $y = 4x^{2.7}$        ii  $y = 3x^{3.5}$

b  i  $y = 5x^{0.6}$        ii  $y = -3x^{0.4}$

c  i  $y = -2x^{-0.7}$        ii  $y = 4x^{-1.5}$

**4** A curve has equation $y = x^2 - 3x^{\frac{1}{2}} + 5$. Find $\dfrac{dy}{dx}$.

**5** Given $f(x) = 4x^3 - 3x^2 + 2x^{-\frac{3}{2}}$, find $f'(x)$.

**6** Find the derivative of the function $f(x) = 12x^{-\frac{1}{3}} + \dfrac{5x^{\frac{2}{5}}}{6}$.

Three students' attempts to differentiate $f(x) = \dfrac{x^2 - 3x}{x^3}$ are shown.

Which is the correct solution? Can you identify the errors made in the incorrect so

| Solution 1 | Solution 2 | Solution |
|---|---|---|
| $f(x) = \dfrac{x^2 - 3x}{x^3}$ | $f(x) = \dfrac{x^2 - 3x}{x^3}$ | $f(x) = \dfrac{x^{2}}{}$ |
| $f'(x) = \dfrac{2x - 3}{3x^2}$ | $= x^{-1} - 3x^{-2}$ $f'(x) = -x^{-2} + 6x^{-3}$ | $= x^{-}$ $f'(x) = -3$ |

## EXERCISE 13D

**1** Find $\dfrac{dy}{dx}$ for the following:

**a** **i** $y = \sqrt[3]{x}$        **ii** $y = \sqrt[5]{x}$

**b** **i** $y = 8\sqrt[4]{x}$        **ii** $y = \dfrac{\sqrt{x}}{3}$

**c** **i** $y = -\dfrac{1}{x}$        **ii** $y = \dfrac{1}{x^4}$

**d** **i** $y = \dfrac{3}{x^2}$        **ii** $y = -\dfrac{2}{5x^{10}}$

e i $y = \dfrac{1}{\sqrt{x}}$

ii $y = \dfrac{1}{\sqrt[3]{x}}$

f i $y = -\dfrac{10}{\sqrt[5]{x}}$

ii $y = \dfrac{8}{3\sqrt[4]{x}}$

**2** Find $f'(x)$ for the following:

a i $f(x) = (2x - 3)(x + 4)$

ii $f(x) = 3x(x - 5)$

b i $f(x) = \sqrt{x}(4x + 3)$

ii $f(x) = \sqrt[3]{x}(x - 1)$

c i $f(x) = \left(\sqrt{x} + 2x\right)^2$

ii $f(x) = \left(\sqrt[4]{x} - 4\right)^2$

d i $f(x) = \left(x + \dfrac{1}{x}\right)^2$

ii $f(x) = \left(x + \dfrac{2}{x}\right)\left(x - \dfrac{2}{x}\right)$

**3** Differentiate the following:

a i $f(x) = \dfrac{3x - 2}{x}$

ii $f(x) = \dfrac{1 + 4x^2}{2x}$

b i $f(x) = \dfrac{\sqrt{x} - 3}{x^2}$

ii $f(x) = \dfrac{x^2 + 4}{\sqrt{x}}$

**4** Differentiate $y = x^2(3x - 4)$.

**5** A curve has equation $y = 2\sqrt{x}\left(x^3 + 4\right)$. Find $y'$.

**6** $y = \sqrt[5]{x^4}$. Find $\dfrac{dy}{dx}$.

**7** Find the derivative of the function $f(x) = \dfrac{8}{3\sqrt[4]{x^3}}$.

**8** A curve has equation $y = \dfrac{3x^5 - 2x}{x^2}$.

a Express $y = \dfrac{3x^5 - 2x}{x^2}$ in the form $y = ax^p + bx^q$.

b Hence find $\dfrac{dy}{dx}$.

**9** $f(x) = \dfrac{(x + 1)(x + 9)}{x}$

Show that $f'(x) = \dfrac{(x - 3)(x + 3)}{x^2}$.

**10** $f(x) = \dfrac{2x^2 + 3}{3\sqrt{x}}$. Find $f'(x)$.

**11** Find the derivative of the curve $y = \dfrac{\left(2\sqrt{x} - 3\right)^2}{\sqrt{x^3}}$.