Section 1: Introduction to vectors

Solutions to Exercise level 2

1. (i) $|\underline{a}| = \sqrt{2^2 + 1^2} = \sqrt{5}$

$$\hat{\underline{a}} = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

(ii) $|\underline{b}| = \sqrt{4^2 + (-3)^2} = 5$

$$\hat{\underline{b}} = \frac{1}{5}\begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

(iii) $|\underline{c}| = \sqrt{(-2)^2 + 5^2} = \sqrt{29}$

$$\hat{\underline{c}} = \frac{1}{\sqrt{29}}\begin{pmatrix} -2 \\ 5 \end{pmatrix}$$

2. (i) $\overrightarrow{AB} = -\underset{\sim}{a} + \underset{\sim}{b} = \underset{\sim}{b} - \underset{\sim}{a}$

$\overrightarrow{AC} = -\underset{\sim}{a} + \underset{\sim}{c} = \underset{\sim}{c} - \underset{\sim}{a}$

(ii) $\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AC} = \frac{1}{2}(\underset{\sim}{c} - \underset{\sim}{a})$

(iii) $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = \underset{\sim}{a} + \frac{1}{2}(\underset{\sim}{c} - \underset{\sim}{a}) = \frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{c}$

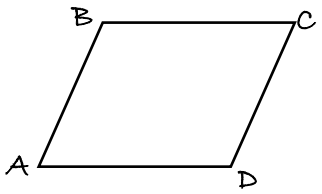
(iv) $\overrightarrow{NM} = \overrightarrow{OM} - \overrightarrow{ON} = \overrightarrow{OM} - \frac{1}{2}\overrightarrow{OB} = \frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{b}$

(v) If N and M coincide, $\overrightarrow{NM} = \underset{\sim}{0}$

$$\frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{b} = \underset{\sim}{0}$$

$$\underset{\sim}{a} + \underset{\sim}{c} = \underset{\sim}{b}$$

3.



$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$$
$$= \overrightarrow{OA} + \overrightarrow{BC}$$
$$= \overrightarrow{OA} + \overrightarrow{OC} - \overrightarrow{OB}$$
$$= \underset{\sim}{a} + \underset{\sim}{c} - \underset{\sim}{b}$$

4. (i) $\overrightarrow{OL} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$

$$= \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \frac{1}{2}\left(\begin{pmatrix} 5 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}\right)$$
$$= \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 3 \\ 4 \end{pmatrix}$$
$$= \begin{pmatrix} 3.5 \\ 5 \end{pmatrix}$$

$\overrightarrow{OM} = \overrightarrow{OB} + \frac{1}{2}\overrightarrow{BC}$

$$= \begin{pmatrix} 5 \\ 7 \end{pmatrix} + \frac{1}{2}\left(\begin{pmatrix} 12 \\ 8 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \end{pmatrix}\right)$$
$$= \begin{pmatrix} 5 \\ 7 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 7 \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} 8.5 \\ 7.5 \end{pmatrix}$$

(ii) $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} 12 \\ 8 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$

$\overrightarrow{LM} = \overrightarrow{OM} - \overrightarrow{OL} = \begin{pmatrix} 8.5 \\ 7.5 \end{pmatrix} - \begin{pmatrix} 3.5 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ 2.5 \end{pmatrix}$

(iii) $AC = \sqrt{(12-2)^2 + (8-3)^2} = \sqrt{100+25} = 5\sqrt{5}$

$LM = \sqrt{(8.5-3.5)^2 + (7.5-5)^2} = \sqrt{25+6.25} = 2.5\sqrt{5}$

(iv) AC and LM are parallel, and LM is half the length of AC.