Readings and Measurements

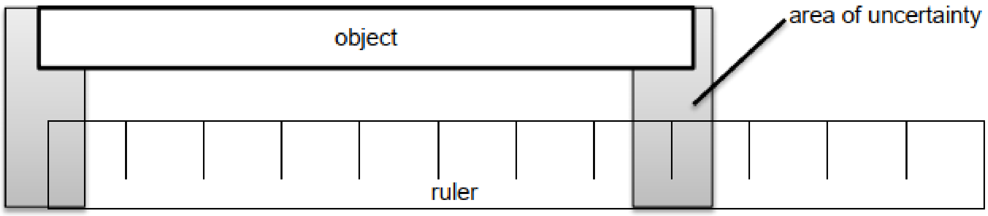
It is useful, when discussing uncertainties, to separate measurements into two forms:
• **readings**: the values found from a single judgement when using a piece of equipment
• **measurements**: the values taken as the difference between the judgements of two values.

Where an instrument has a **fixed/calibrated zero position** we class this as a **reading** as we only have to make one judgement at the end point, but an instrument where **we place the zero** (i.e. a ruler) classes as a **measurement** as we must take **two judgements** (the zero mark, and our determined end point). You can use the following table as a guide.

Reading (one judgement only)	Measurement (two judgements required)
thermometer	ruler
top pan balance	vernier calliper
measuring cylinder	micrometer
digital voltmeter	protractor
Geiger counter	stopwatch
pressure gauge	analogue meter

Each **reading** taken has an uncertainty of ± half of the precision of the instrument. For example, a thermometer with graduations which are $1\ °C$ apart as an uncertainty on each reading of $\pm 0.5\ °C$ – this occurs because we can make a judgement as to which graduation the meniscus of the fluid in thermometer is nearest to, but can't judge actual values between graduations.

Each **measurement** such as a length using a ruler requires **two readings**. Each reading contributes an uncertainty of ± half of the precision of the instrument, so the actual uncertainty of a measurement will be ± the precision of the instrument. For example;



If the graduations on the ruler are every $1\ mm$, then each reading (once at the zero mark and once at the end point of the object) contributes an uncertainty of $\pm 0.5\ mm$, hence the uncertainty on the length of the object is $\pm 1\ mm$.

Measuring a **change in length** is an example of a difference between **two measurements**. If we were to do this using the ruler above, each **measurement** contributes an uncertainty of $\pm 1\ mm$, so the uncertainty would be $\pm\ 2\ mm$.

In summary;

- The uncertainty of a reading (one judgement) is at least ±0.5 of the smallest scale reading.
- The uncertainty of a measurement (two judgements) is at least ±1 of the smallest scale reading
- The uncertainty of a difference between two values is twice that of the uncertainty of a single measurement on that specific instrument.

Other Uncertainty Rules

Sometimes, our uncertainties arrive from sources other than just the precision of instruments. Examples of this are reaction times, judgment of the position of an object, being unable to straighten a wire when attempting to measure its length. Here are some useful suggestions for obtaining uncertainties in these values;

Given Values
In all such cases assume the uncertainty to be ± 1 in the last significant digit.
The value of the charge on the electron is stated as $1.60 \times 10^{-19}\ C$ so the uncertainty will be $\pm\ 0.01 \times 10^{-19} C$.

Measuring over multiple instances
Sometimes we will measure over multiple instances of an action to reduce uncertainty. Common examples include timing several swings of a pendulum, measuring over several fringes on an interference pattern, measuring the thickness of several sheets of paper together rather than one sheet.
In these cases, we can divide the uncertainty by the number of instances just as we would for the actual value. For example;
Time taken for pendulum to swing 10 times: $5.1 \pm 0.1\ s$, so Mean time taken for one swing: $0.51 \pm 0.01\ s$.

Repeating measurements
This is a way we can reduce uncertainty in a measurement.
If we repeat a measurement, we can identify anomalous readings before calculating a mean. The uncertainty is then calculated as half the range of the measured values.

Percentage uncertainties
So far, all of the uncertainties shown are **absolute uncertainties** and have units matching the measurements they are attached to. However, when we move to the idea of calculations and combining uncertainties from different quantities we must often use **percentage uncertainties.**

The percentage uncertainty in a measurement can be calculated using:

$$percentage\ uncertainty = \frac{uncertainty}{value} \times 100\%$$

The percentage uncertainty in a repeated measurement can also be calculated using:

$$percentage\ uncertainty = \frac{uncertainty}{mean\ value} \times 100\%$$

Error bars

When plotting a graph, we should include error bars on the values of our dependant variable. These are a good way of visualising the uncertainty of a particular point. Follow these simple rules;

- Plot the data point at the mean value
- Calculate the range of the data, ignoring any anomalies
- Add error bars with lengths equal to half of the range on either side of the data point.

The error bars then make it easier to judge anomalous readings once the line of best fit has been drawn.

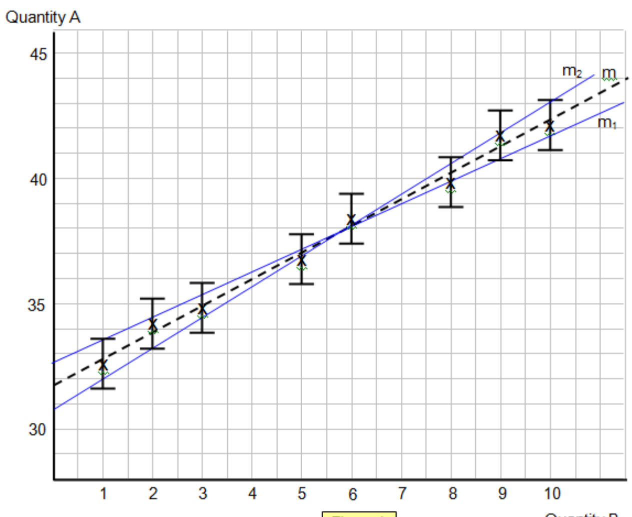


Figure 1

Uncertainties from Gradients

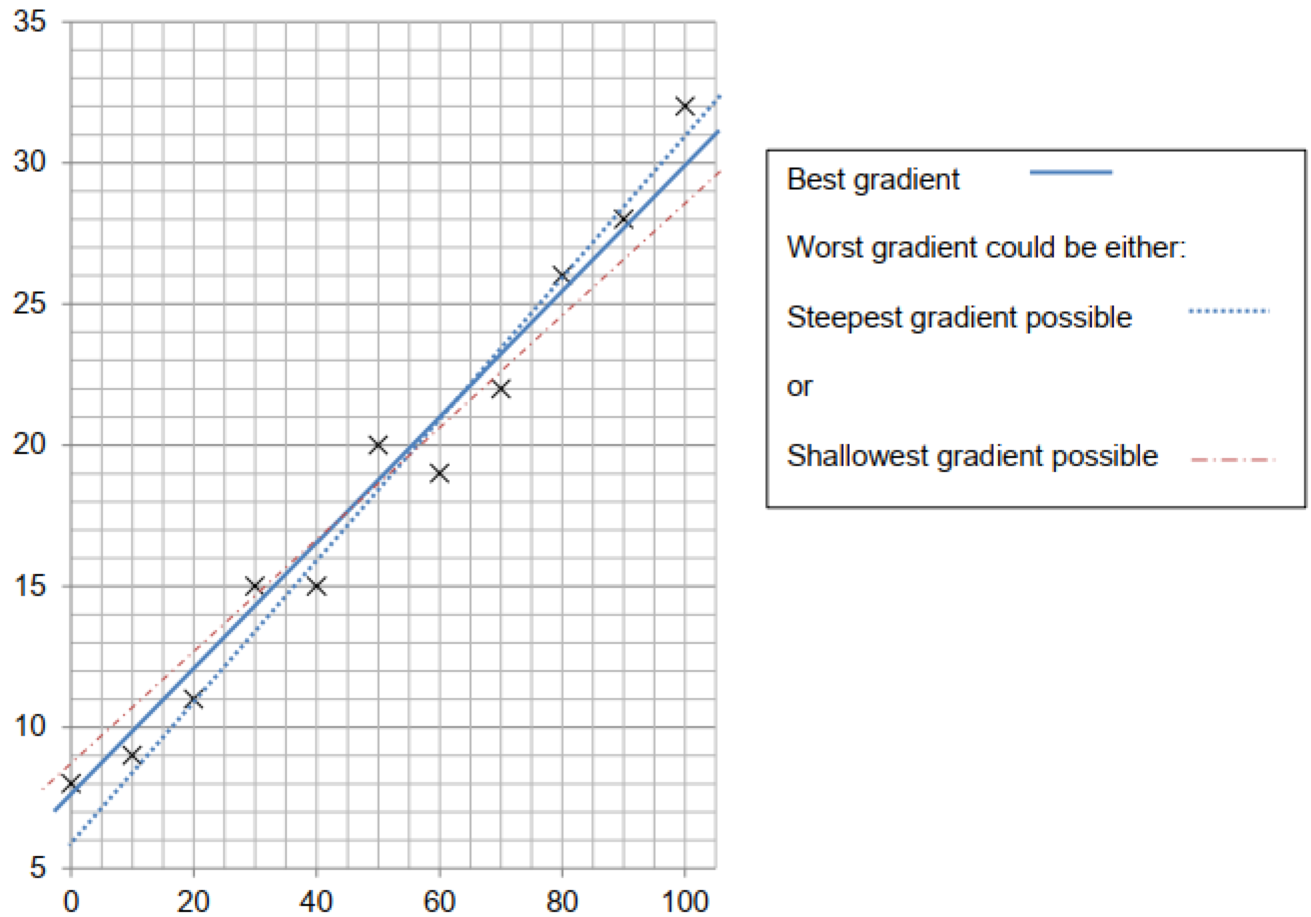
You will often be using gradients in the final stages of required experiments in order to calculate a desired value. In these instances, we will usually use the percentage uncertainty in the gradient to find the percentage uncertainty in the desired value (the rules for how to do this are on the next page). However, you can follow the guide below to find the percentage uncertainty in your gradient.

To find the uncertainty in a gradient, two lines should be drawn on the graph. One should be the "best" line of best fit. The second line should be the steepest or shallowest gradient line of best fit possible from the data. The gradient of each line should then be found.

The uncertainty in the gradient is found by:

$$\text{percentage uncertainty} = \frac{|\text{best gradient} - \text{worst gradient}|}{\text{best gradient}} \times 100\%$$

Note the modulus bars meaning that this percentage will always be positive.



In the same way, the percentage uncertainty in the y-intercept can be found:

$$\text{percentage uncertainty} = \frac{|\text{best } y \textit{ intercept} - \text{worst } y \textit{ intercept}|}{\text{best } y \textit{ intercept}} \times 100\%$$

Combining uncertainties

Follow the rules below when performing any calculations with measured values.

Combination	Operation	Example
Adding or subtracting values $a = b + c$	Add the absolute uncertainties $\Delta a = \Delta b + \Delta c$	Object distance, u = (5.0 ± 0.1)cm Image distance, v = (7.2 ± 0.1)cm Difference ($v – u$) = (2.2 ± 0.2)cm
Multiplying values $a = b \times c$	Add the percentage uncertainties $\varepsilon a = \varepsilon b + \varepsilon c$	Voltage = (15.20 ± 0.1) V Current = (0.51 ± 0.01) A Percentage uncertainty in voltage = 0.7% Percentage uncertainty in current = 1.96% Power = Voltage × current = 7.75 W Percentage uncertainty in power = 2.66% Absolute uncertainty in power = ± 0.21 W
Dividing values $a = \frac{b}{c}$	Add the percentage uncertainties $\varepsilon a = \varepsilon b + \varepsilon c$	Mass of object = (30.2 ± 0.1) g Volume of object = (18.0 ± 0.5)cm^3 Percentage uncertainty in mass of object = 0.3 % Percentage uncertainty in volume = 2.8% Density = $\frac{30.2}{18.0}$ = 1.68 g cm^{-3} Percentage uncertainty in density = 3.1% Absolute uncertainty in density = ± 0.05 g cm^{-3}
Power rules $a = b^c$	Multiply the percentage uncertainty by the power $\varepsilon a = c \times \varepsilon b$	Radius of circle = (6.0 ± 0.1)cm Percentage uncertainty in radius = 1.6% Area of circle = πr^2 = 113.1cm^2 Percentage uncertainty in area = 3.2% Absolute uncertainty = ± 3.6cm^2 (Note – the uncertainty in π is taken to be zero)

Note: Absolute uncertainties (denoted by Δ) have the same units as the quantity.

Percentage uncertainties (denoted by ε) have no units.

Uncertainties in trigonometric and logarithmic functions will not be tested in A-level exams.