A Level Further Mathematics Preparation Work

We are delighted that you have chosen to study A Level Further Maths. You will study the Single Maths content first and then move onto the Further Maths content after Christmas. Therefore it is vital that you are confident with all the material provided for A Level Mathematics and this should be your priority in preparing for starting in September. Please ensure you work through the content on A Level Maths document up to section 3 and bring it in September. You shouldn't have too many problems so make sure you give yourself time to enjoy section 4!

To be successful in Further Mathematics, you need to be prepared to work hard, think quickly and to undertake a lot of independent study. You need to be good at problem solving and using your knowledge to think hard about less familiar problems.

Please attempt these problems and bring your solutions to your first Maths lessons in September. You might find some of them too tricky but have a go! Your teacher will be most interested in looking at your thinking, your methods and your layout rather than you necessarily achieving solutions.

a) Prove that $4(p-3)-2(2p-1)$ is always a negative number.

b) Prove that $8(y+3)+3(2-y)$ is a multiple of 5 when y is a positive number.

c) a is a positive integer.

Prove that $4a^2(2a+1)-(2a)^2$ is a cube number.

d) a and b are positive integers. $a<b$.

Prove that $\frac{ax+3a}{bx+3b}<1$ $\quad x\neq -3$

e) Express $x^2+6x+11$ in the form $(x+a)^2+b$ where a and b are integers.

Hence, prove that $x^2+6x+11$ is always positive.

f) $f(x)=(2x+3)^2+8(x+2)$ for all values of x.

Prove that there is exactly one value of x for which $f(x)=0$.

g) The nth term of a sequence is $\frac{1}{2}n(n+1)$

Work out an expression for the $(n-1)$th term of the sequence.

Give you answer in its simplest form.

Hence, or otherwise, prove that the sum of any consecutive pair of terms of the sequence is a square number.

h) Prove that $\frac{x^2-4}{5x-10}\times\frac{10x^2}{x+2}$ is always positive.

i) The nth term of a sequence is $\frac{3n}{5n+12}$

Work out the position of the term that has a value of $\frac{1}{2}$.

j) Solve $(3-\sqrt{x})^{\frac{1}{3}}=-2$

k) $\sqrt[4]{x}=2$ and $y^{-2}=25$

$x>0$ and $y<0$

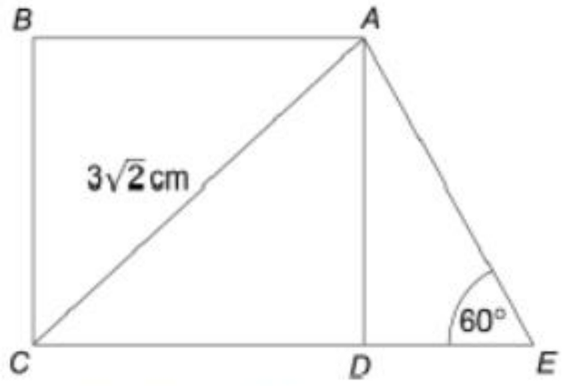
Work out the value of $\frac{x}{y}$

l) Solve $\sqrt{125}+\sqrt{20}=\sqrt{80}+\sqrt{x}$

m) $ABCD$ is a square.

CDE is a straight line.

AC is $3\sqrt{2}$cm and angle DEA is 60^o



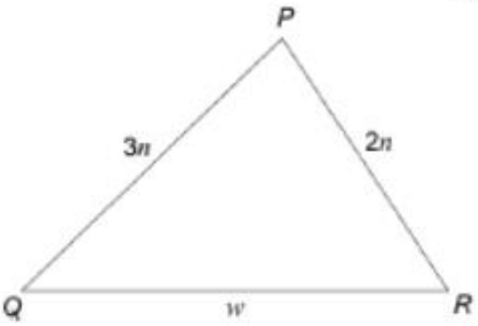
Not drawn accurately

Show that the side of the square is 3cm

Show that the perimeter of trapezium $ABCE$ is $3(3+\sqrt{3})$cm.

n) In triangle PQR, $\cos P = \frac{1}{3}$

Not drawn accurately



Show that triangle PQR is isosceles.

Once you've had a go, reflect on the following:

- Which was your favourite problem and why?
- Which has the most elegant solution?
- Which is the most deceptive problem?
- Which problem is the most challenging?
- Which can you re-write using different numbers but the same structure?
- Which can you re-write using the same surface but a different underlying problem?

Oxford University entrance problems – multiple choice (choose one answer)

B. The product of a square number and a cube number is

(a) always a square number, and never a cube number.
(b) always a cube number, and never a square number.
(c) sometimes a square number, and sometimes a cube number.
(d) never a square number, and never a cube number.
(e) always a cube number, and always a square number.

C. Let a, b, c and d be real numbers. The two curves $y = ax^2 + c$ and $y = bx^2 + d$ have exactly two points of intersection precisely when

(a) $\frac{a}{b} < 1$, (b) $\frac{a}{b} < \frac{c}{d}$, (c) $a < b$, (d) $c < d$, (e) $(d-c)(a-b) > 0$.

D. If $f(x) = x^2 - 5x + 7$, what are the coordinates of the minimum of $y = f(x-2)$?

(a) $\left(\frac{5}{2}, \frac{3}{4}\right)$, (b) $\left(\frac{9}{2}, \frac{3}{4}\right)$, (c) $\left(\frac{1}{2}, \frac{3}{4}\right)$, (d) $\left(\frac{9}{2}, \frac{-5}{4}\right)$, (e) $\left(\frac{5}{2}, \frac{-5}{4}\right)$.

Cambridge University Entrance Problem

Find the integer, n, that satisfies $n^2 < 33127 < (n+1)^2$. Find also a small integer m such that $(n+m)^2 - 33127$ is a perfect square. Hence express 33127 in the form pq, where p and q are integers greater than 1.

By considering the possible factorisations of 33127, show that there are exactly two positive values of m for which $(n+m)^2 - 33127$ is a perfect square, and find the other value.