

AS and A LEVEL
**MATHEMATICS A
AND MATHEMATICS B (MEI)**
Student Guide

Bridging the gap between GCSE and AS/A Level Mathematics

The object of these exercises is to help you get started with the A Level course, and to smooth your path through it. Many students find A Level a challenge compared with GCSE. This is a recognised issue – if you find this you are very far from being alone! I hope that these pages will help you.

The main focus is on developing skills, as opposed to learning new material. So I suggest that you don't approach it with the mind-set "what do I have to do to get full marks?" but "what can I learn that will help me in my future studies?" You want to be fluent in a number of aspects of GCSE work – not just able to get an answer that would score a mark in a GCSE examination. For example, you will want to get a result in a form that you yourself can go on and use readily. So there will be an emphasis on real fluency in algebra. Likewise, you should try to develop the ability to think of the shapes of graphs corresponding to algebraic formulae. Do not be afraid of any of this; over the years the vast majority of A Level students have succeeded with the course. But you will find the path easier, enjoy it more and be more confident if you are really on top of the basic vocabulary of the subject before you start putting it together into A Level sentences.

One recurring theme here is; just because you *can* multiply out brackets, it doesn't mean that you *should*. Indeed, at this level it is often better to keep an expression in its factorised (bracketed) form.

Good luck!

This booklet is only a collection of exercises – for full notes and examples on these topics (as well as the answers!) go to

<https://mathematics.bourne-grammar.lincs.sch.uk/>

Further reading

There are not many books designed for the sort of transition that this booklet represents, but an outstanding exception is:

Fyfe, M. T., Jobbings, A. and Kilday, K. (2007) *Progress to Higher Mathematics*, Arbelos.

ISBN 9780955547706.

Alternatively, also published by Arbelos is an expansion of the same book which is more specifically aimed at the transition to A Level. You may not need quite so much as this:

Fyfe, M. T., and Kilday, K. (2011) *Progress to Advanced Mathematics*, Arbelos.

ISBN 9780955547737.

OCR Resources: *the small print*

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Exercise 1.1 Simple Algebraic Expressions

1 Find the values of the letters p , q and r that make the following pairs of expressions always equal.

(a) $\frac{1}{7}x = \frac{x}{p}$ (b) $\frac{1}{5}(2x+3) = \frac{(2x+3)}{q}$ (c) $\frac{3}{10}(2-7x) = \frac{3(2-7x)}{r}$

2 Solve the following equations.

(a) $\frac{60}{x+4} = 12$ (b) $\frac{35}{2x-3} = 5$ (c) $\frac{20}{6-x} = \frac{1}{2}$

3 Make $\cos C$ the subject of the formula $c^2 = a^2 + b^2 - 2ab \cos C$.

4 (a) Multiply $\frac{x+5}{4}$ by 8. (b) Multiply $(x+2) \div 3$ by 12.

(c) Multiply $\frac{1}{2}(x+7)$ by 6. (d) Multiply $\frac{1}{4}(x-3)$ by 8.

5 Solve the following equations.

(a) $\frac{3}{4}(2x+3) = \frac{5}{8}(x-2)$ (b) $\frac{1}{6}(5x+11) = \frac{2}{3}(2x-4)$

(c) $\frac{5}{9}(3x+1) = \frac{7}{12}(2x+1)$

6 Make x the subject of the following equations.

(a) $\frac{a}{b}(cx+d) = x+2$ (b) $\frac{a}{b}(cx+d) = \frac{2a}{b^2}(x+2d)$

7 Simplify the following as far as possible.

(a) $\frac{a+a+a+a+a}{5}$ (b) $\frac{b+b+b+b}{b}$

(c) $\frac{c \times c \times c \times c \times c}{c}$ (d) $\frac{d \times d \times d \times d}{4}$

Exercise 1.2 Algebraic Fractions

1 Work out the following. Answers may be left as improper fractions.

(a) $\frac{4}{7} \times 5$ (b) $\frac{5}{12} \times 3$ (c) $\frac{7}{9} \times 2$ (d) $\frac{4}{15} \times 3$

(e) $\frac{8}{11} \div 4$ (f) $\frac{8}{11} \div 3$ (g) $\frac{6}{7} \div 3$ (h) $\frac{6}{7} \div 5$

(i) $\frac{3x}{y} \times x$ (j) $\frac{3x}{y^2} \times y$ (k) $\frac{5x^3}{4y} \div x$ (l) $\frac{5x^2}{6y} \div y$

(m) $\frac{5x^3}{2y} \times 3x$ (n) $\frac{3y^4}{4x^2z} \times 2x$ (o) $\frac{6x^2y^3}{5z} \div 2xy$ (p) $\frac{5a^2}{6x^3z^2} \div 2y$

2 Make x the subject of the following formulae.

(a) $\frac{1}{2}A = \pi x^2$ (b) $V = \frac{4}{3}\pi x^3$ (c) $\frac{1}{2}(u + v) = tx$ (d) $W = \frac{2}{3}\pi x^2h$

3 Simplify the following compound fractions.

(a) $\frac{\frac{1}{x} + 1}{\frac{1}{x} + 3}$ (b) $\frac{\frac{2}{x} + 1}{\frac{3}{x} - 1}$ (c) $\frac{\frac{1}{x+1} + 2}{\frac{1}{x+1} + 1}$

4 Write as single fractions.

(a) $\frac{2}{x-1} + \frac{1}{x+3}$ (b) $\frac{2}{x-3} - \frac{1}{x+2}$ (c) $\frac{1}{2x-1} - \frac{1}{3x+2}$ (d) $\frac{3}{x+2} + 1$ (e)

$2 - \frac{1}{x-1}$ (f) $\frac{2x}{x+1} - 3$ (g) $\frac{3}{4(2x-1)} - \frac{1}{4x^2-1}$

5 Write as single fractions.

(a) $\frac{x+1}{\sqrt{x}} + \sqrt{x}$ (b) $\frac{2x}{\sqrt{x+3}} + \sqrt{x+3}$ (c) $\frac{x}{\sqrt[3]{x-2}} + \sqrt[3]{(x-2)^2}$

6 Write the following in the form $1 + \frac{a}{x+b}$.

(a) $\frac{x+1}{x-5}$ (b) $\frac{x+3}{x+1}$ (c) $\frac{x+2}{x+5}$ (d) $\frac{x-6}{x-2}$

7 Write the following equations without fractions. (A , B etc. are constants that remain in your answers.)

$$(a) \quad \frac{1}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

$$(b) \quad \frac{x+2}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3}$$

$$(c) \quad \frac{2}{(x+1)(x+2)(x-3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-3}$$

$$(d) \quad \frac{1}{(x-2)^2(x+1)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+1}$$

$$(e) \quad \frac{1}{x^2(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$$

Exercise 1.3 Quadratic Expressions

1 Write without brackets.

(a) $(x + 5)^2$ (b) $(x - 4)^2$ (c) $(2x + 1)^2$

(d) $(3x - 2)^2$ (e) $(x + 2)(x - 2)$ (f) $(3x + 4)(3x - 4)$

2 Simplify the following equations into the form $ax + by + c = 0$.

(a) $(x + 3)^2 + (y + 4)^2 = (x - 2)^2 + (y - 1)^2$

(b) $(x + 5)^2 + (y + 2)^2 = (x - 5)^2 + (y - 2)^2$

(c) $(2x + 1)^2 + (y - 3)^2 = (2x + 3)^2 + (y + 1)^2$

3 Simplify the following where possible.

(a) $\sqrt{x^2 + 4}$ (b) $\sqrt{x^2 - 4x + 4}$ (c) $\sqrt{x^2 - 1}$

(d) $\sqrt{x^2 + 9x}$ (e) $\sqrt{x^2 - y^2}$ (f) $\sqrt{x^2 + 2xy + y^2}$

4 Write the following in the form $(x + a)^2 + b$.

(a) $x^2 + 8x + 19$ (b) $x^2 - 10x + 23$ (c) $x^2 + 2x - 4$

(d) $x^2 - 4x - 3$ (e) $x^2 - 3x + 2$ (f) $x^2 - 5x - 6$

5 Write the following in the form $a(x + b)^2 + c$.

(a) $3x^2 + 6x + 7$ (b) $5x^2 - 20x + 17$ (c) $2x^2 + 10x + 13$

6 Write the following in the form $(ax + b)^2 + c$.

(a) $4x^2 + 12x + 14$ (b) $9x^2 - 12x - 1$ (c) $16x^2 + 40x + 22$

7 Factorise as fully as possible.

(a) $x^2 - 25$

(b) $4x^2 - 36$

(c) $4x^2 - 9y^4$

(d) $3x^2 - 7x + 2$

(e) $3x^2 - 5x + 2$

(f) $6x^2 - 5x - 6$

(g) $8x^2 - 2x - 15$

8 Multiply out and simplify.

(a) $\left(x + \frac{1}{x}\right)^2$

(b) $\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

(c) $\left(x + \frac{2}{x}\right)\left(x - \frac{3}{x}\right)$

Exercise 1.4 Cancelling

1 Simplify the following as far as possible.

(a) $5x + 3y + 7x - 3y$ (b) $3x^2 + 4xy + y^2 + x^2 - 4xy - y^2$.

(c) $\frac{4+6x}{2}$ (d) $\frac{4 \times 6x}{2}$ (e) $\frac{3x+xy}{x}$

(f) $\frac{3x \times xy}{x}$ (g) $\frac{4x+10y}{8x+6y}$ (h) $\frac{3x-6y}{9x-3y}$

(i) $\frac{4x+9y}{2x+3y}$ (j) $\frac{4x+6y}{6x+9y}$ (k) $\frac{5xy+6y^2}{10x+12y}$

(l) $\frac{3x^2+4y^2}{6x^2-8y^2}$ (m) $\frac{x-3}{3-x}$ (n) $\frac{x^2-2xy-y^2}{y^2+2xy-x^2}$

2 Make x the subject of the following formulae.

(a) $\frac{ax}{b} = \frac{py}{qz}$ (b) $\frac{3\pi ax}{b} = \frac{4y^2}{qz}$

3 Simplify the following.

(a) $\frac{2\pi x}{ab} \div \frac{1}{3}\pi r^3$ (b) $\frac{2\pi h^2}{rb} \div \frac{4}{3}\pi hr^2$

4 Simplify into a single factorised expression.

(a) $(x-3)^2 + 5(x-3)^3$ (b) $4x(2x+1)^3 + 5(2x+1)^4$

(c) $\frac{1}{2}k(k+1) + (k+1)$ (d) $\frac{1}{6}k(k+1)(2k+1) + (k+1)^2$

5 Simplify as far as possible.

(a) $\frac{x^2+6x+8}{x^2-x-6}$ (b) $\frac{3x^2-2x-8}{x^2-4}$

(c) $\frac{(x+3)^2-2(x+3)}{x^2+2x-3}$ (d) $\frac{x(2x-1)^2-x^2(2x-1)}{(x-1)^2}$

(e) $\frac{\frac{x^2}{\sqrt{x^2+1}} - \sqrt{x^2+1}}{x^2}$ (f) $-\frac{\frac{x}{2\sqrt{1-x}} + \sqrt{1-x}}{x^2}$

(g) $\frac{\frac{\sqrt{x}}{2\sqrt{1+x}} - \frac{\sqrt{1+x}}{2\sqrt{x}}}{x}$ (h) $\frac{\sqrt[3]{1+x} - \frac{x}{3\sqrt[3]{(1+x)^2}}}{\sqrt[3]{1+x}}$

Exercise 1.5 Simultaneous Equations

Solve the following simultaneous equations.

1 $x^2 + xy = 12$

$$3x + y = 10$$

2 $x^2 - 4x + y^2 = 21$

$$y = 3x - 21$$

3 $x^2 + xy + y^2 = 1$

$$x + 2y = -1$$

4 $x^2 - 2xy + y^2 = 1$

$$y = 2x$$

5 $c^2 + d^2 = 5$

$$3c + 4d = 2$$

6 $x + 2y = 15$

$$xy = 28$$

7 $2x^2 + 3xy + y^2 = 6$

$$3x + 4y = 1$$

8 $2x^2 + 4xy + 6y^2 = 4$

$$2x + 3y = 1$$

9 $4x^2 + y^2 = 17$

$$2x + y = 5$$

10 $2x^2 - 3xy + y^2 = 0$

$$x + y = 9$$

11 $x^2 + 3xy + 5y^2 = 15$

$$x - y = 1$$

12 $xy + x^2 + y^2 = 7$

$$x - 3y = 5$$

13 $x^2 + 3xy + 5y^2 = 5$

$$x - 2y = 1$$

14 $4x^2 - 4xy - 3y^2 = 20$

$$2x - 3y = 10$$

15 $x^2 - y^2 = 11$

$$x - y = 11$$

16 $\frac{12}{x} + \frac{1}{y} = 3$

$$x + y = 7$$

Exercise 1.6 Fractional and negative powers, surds

1 Write the following as powers of x .

(a) $\frac{1}{x}$ (b) $\frac{1}{x^5}$ (c) $\sqrt[5]{x}$ (d) $\sqrt[3]{x^5}$ (e) $\frac{1}{\sqrt{x}}$ (f) $\frac{1}{\sqrt{x^3}}$

2 Write the following without negative or fractional powers.

(a) x^{-4} (b) x^0 (c) $x^{1/6}$ (d) $x^{3/4}$ (e) $x^{-3/2}$

3 Write the following in the form ax^n .

(a) $4\sqrt[3]{x}$ (b) $\frac{3}{x^2}$ (c) $\frac{5}{\sqrt{x}}$ (d) $\frac{1}{2x^3}$ (e) 6

4 Write as sums of powers of x .

(a) $x^3\left(x + \frac{1}{x}\right)$ (b) $\frac{x^4+1}{x^2}$ (c) $x^{-5}\left(x + \frac{1}{x^2}\right)$

5 Write the following in surd form.

(a) $\sqrt{75}$ (b) $\sqrt{180}$ (c) $\frac{12}{\sqrt{6}}$ (d) $\frac{1}{\sqrt{5}}$ (e) $\frac{3}{\sqrt{12}}$

6 Rationalise the denominators in the following expressions.

(a) $\frac{1}{\sqrt{2}-1}$ (b) $\frac{2}{\sqrt{6}-2}$ (c) $\frac{6}{\sqrt{7}+2}$

(d) $\frac{1}{3+\sqrt{5}}$ (e) $\frac{1}{\sqrt{6}-\sqrt{5}}$

7 Simplify $\frac{1}{\sqrt{2}+\sqrt{1}} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \dots + \frac{1}{\sqrt{100}+\sqrt{99}}$.

Exercise 2.1 Trigonometric Equations

1 Solve the following equations for $0 \leq x < 360$. Give your answers to the nearest 0.1° .

(a) $\sin x^\circ = 0.9$ (b) $\cos x^\circ = 0.6$ (c) $\tan x^\circ = 2$

(d) $\sin x^\circ = -0.4$ (e) $\cos x^\circ = -0.5$ (f) $\tan x^\circ = -3$

2 Solve the following equations for $-180 \leq x < 180$. Give your answers to the nearest 0.1° .

(a) $\sin x^\circ = 0.9$ (b) $\cos x^\circ = 0.6$ (c) $\tan x^\circ = 2$

(d) $\sin x^\circ = -0.4$ (e) $\cos x^\circ = -0.5$ (f) $\tan x^\circ = -3$

3 Solve the following equations for $0 \leq x < 360$. Give your answers to the nearest 0.1° .

(a) $\sin 2x^\circ = 0.829$ (b) $\cos 3x^\circ = 0.454$ (c) $\tan 4x = 2.05$

(d) $\sin \frac{1}{2}x^\circ = 0.8$ (e) $\cos \frac{1}{2}x^\circ = 0.3$ (f) $\tan \frac{1}{3}x^\circ = 0.7$

Exercise 2.2 Other Trigonometric Methods

Do not use a calculator in this exercise.

1 In this question θ is in the range $0 \leq \theta < 90$.

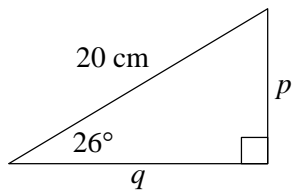
(a) Given that $\sin \theta = \frac{12}{13}$, find the exact values of $\cos \theta$ and $\tan \theta$.

(b) Given that $\tan \theta = \frac{6}{7}$, find the exact values of $\sin \theta$ and $\cos \theta$.

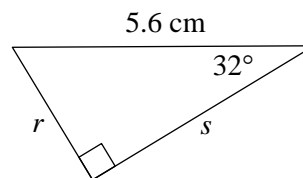
(c) Given that $\cos \theta = \frac{5}{8}$, find the exact values of $\sin \theta$ and $\tan \theta$.

2 Find expressions, of the form $a \sin \theta$ or $b \cos \theta$, for the sides labelled with letters in these triangles.

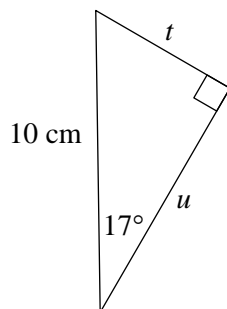
(a)



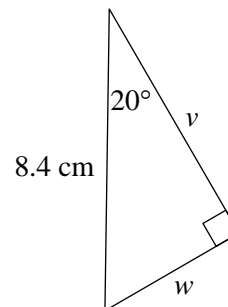
(b)



(c)



(d)



Exercise 3.1 Straight line graphs

1 Rearrange the following in the form $ax + by + c = 0$ or $ax + by = c$ as convenient, where a , b and c are integers and $a > 0$.

(a) $y = 3x - 2$

(b) $y = \frac{1}{2}x + 3$

(c) $y = -\frac{3}{4}x + 3$

(d) $y = \frac{7}{2}x - \frac{5}{4}$

(e) $y = -\frac{2}{3}x + \frac{3}{4}$

(f) $y = \frac{4}{7}x - \frac{2}{3}$

2 Rearrange the following in the form $y = mx + c$. Hence find the gradient and the y -intercept of each line.

(a) $2x + y = 8$

(b) $4x - y + 9 = 0$

(c) $x + 5y = 10$

(d) $x - 3y = 15$

(e) $2x + 3y + 12 = 0$

(f) $5x - 2y = 20$

(g) $3x + 5y = 17$

(h) $7x - 4y + 18 = 0$

3 Sketch the following lines. Show on your sketches the coordinates of the intercepts of each line with the x -axis and with the y -axis.

(a) $2x + y = 8$

(b) $x + 5y = 10$

(c) $2x + 3y = 12$

(d) $3x + 5y = 30$

(e) $3x - 2y = 12$

(f) $4x + 5y + 20 = 0$

Exercise 3.2 Basic shapes of curved graphs

Sketch (do not *plot*) the general shape of the graphs of the following curves.

Axes are not required but can be included in the questions marked with an asterix.

1 $y = x^2 - 3x + 2$

2 $y = -x^2 + 5x + 1$

3 $y = 1 - x^2$

4 $y = (x - 2)(x + 4)$

5 $y = (3 - x)(2 + x)$

6 $y = (1 - x)(5 - x)$

7 $y = x^3$

8 $y = -x^3$

9* $y = \frac{3}{x}$

10* $y = -\frac{2}{x}$

11 $y = (x - 2)(x - 3)(x + 1)$

12* $y = \frac{2}{x^2}$

13 Sketch on the same axes the general shape of the graphs of $y = x^2$ and $y = x^4$.

14 Sketch on the same axes the general shape of the graphs of $y = x^3$ and $y = x^5$.

Exercise 3.3 Factors

Sketch the curves in questions 1–21. Use a different diagram for each. Show the x -coordinates of the intersections with the x -axis.

1 $y = x^2$

2 $y = (x - 1)(x - 3)$

3 $y = (x + 2)(x - 4)$

4 $y = x(x - 3)$

5 $y = (x + 2)(3x - 2)$

6 $y = x(4x + 3)$

7 $y = -x(x - 3)$

8 $y = (2 - x)(x + 1)$

9 $y = (3 - x)(2 + x)$

10 $y = (x + 2)(x - 1)(x - 4)$

11 $y = x(x - 1)(x + 2)$

12 $y = -x(x - 1)(x + 2)$

13 $y = (3 - x)(2 - x)(1 - x)$

14 $y = (x - 1)^2(x - 3)$

15 $y = (x - 1)(x - 3)^2$

16 $y = (x + 1)^3$

17 $y = (2 - x)(x + 1)^3$

18 $y = (x + 1)(x + 2)(x - 1)(x - 2)$

19 $y = -(x + 3)(x + 2)(x - 1)(x - 4)$

20 $y = (x - 2)^2(x + 2)^2$

21 $y = (x - 1)(x - 2)^2(x - 3)^3$

22 (a) Sketch the graph of $y = x^2$.

(b) Sketch $y = 2x^2$ on the same axes.

(c) Sketch $y = x^2 + 1$ on the same axes.

23 (a) Sketch the graph of $y = \sqrt{x}$.

(b) Sketch $y = 2\sqrt{x}$ on the same axes.

24 (a) Sketch the graph of $y = \frac{1}{x}$.

(b) Sketch $y = \frac{1}{x} + 1$ on the same axes.

25 (a) Sketch the graph of $y = \frac{1}{x^2}$.

(b) Sketch $y = \frac{2}{x^2}$ on the same axes.

26 (a) Sketch the graph of $y = x^3$.

(b) Sketch $y = 2x^3$ on the same axes.

- 27** (a) Sketch the graph of $y = x^4$.
- (b) Sketch $y = 3x^4$ on the same axes.
- 28** (a) Sketch the graph of $y = x^3 - 4x$.
- [Hint: It cuts the x -axis at -2 , 0 and 2 .]
- (b) Sketch $y = 2x^3 - 8x$ on the same axes.
- 29** (a) Sketch the graph of $y = x^4 - x^2$.
- [Hint: It cuts the x -axis at 1 and -1 , and touches the axis at 0 .]
- (b) Sketch $y = -x^4 + x^2$ on the same axes.
- 30** Sketch, on separate axes, the following graphs. Show the x -coordinates of the intersections with the x -axis.
- (a) $y = 4 - x^2$
- (b) $y = (x - 2)(x + 1)$
- (c) $y = -(x - 2)(x + 1)$
- (d) $y = x(x + 4)$
- (e) $y = (x - 2)^2$
- (f) $y = -(x + 1)^2$
- (g) $y = (1 - x)(2 + x)$